Physics I
ISI B.Math
Midterm Exam : February 24, 2016

Total Marks: 70

Answer question 1 and any 4 of the rest

1.(**Marks** $= \mathbf{2} \times \mathbf{7} = \mathbf{14}$)

In this question you need to write down the correct option. No explanation is necessary. There is only one correct option.

(i)A particle is moving under the influence of a force $\mathbf{F} = (\frac{a \sin t}{r^2} - \frac{b \cos t}{r^3})\hat{\mathbf{r}}$, where a and b are constants. Which of the following statements are not true about the motion of the particle ?

(a) Angular momentum is conserved
(b) Total mechanical energy is conserved
(c) $\nabla \times \mathbf{F} = 0$.
(d) The work done by the particle in moving from one point to another is independent of path.
(e) The motion remains confined to a plane.

(ii) An undamped harmonic oscillator of mass m and angular frequency ω_0 moves in one dimension along the x- axis. If we plot x vs the linear momentum p_x (a phase space plot) for a given set of initial conditions, the resulting curve will be

(a) closed
(b) open
(c) can be closed or open depending upon the initial conditions.

(iii) S is an inertial frame S' is another inertial frame moving with velocity $\mathbf{v}$ with respect to S. For which of the following quantities will an observer in S and one is S' disagree on their measurements ?

(a) The gravitational force $\mathbf{F}$ acting on a particle of mass m_1 due to another particle m_2 .
(b) The mutual potential energy $U(\mathbf{r})$ of the above two particles interacting though a gravitational force where $\mathbf{r}$ is the relative position vector between the two particles.
(c) The total mechanical energy (kinetic + potential) of the two particles mentioned in the two above options.

(iv) A particle is moving in three dimensions under the influence of a potential $U(r) = \frac{1}{2}kr^2$, where k is a positive constant. Which of the statements about the motion of the particle is false ?

(a) A possible trajectory for the particle is a circular orbit.
(b) The particle can have bounded or unbounded motion depending on its total energy.
(c)The radius vector of the particle sweeps out equal areal in equal times
(d) The total energy of the particle is conserved.

(v) A ball bearing of mass m is released from rest in a vertical column of castor oil which exerts a

retarding force equal to $-mkv$ on the ball bearing. which of the following expressions can correctly describe its velocity at time t ?

(a) $v = \frac{g}{k}(1 - e^{-}kt)$
(b) $v = \frac{g}{k}(1 - e^{kt})$
(c) $v = \frac{g}{k}e^{-}kt$
(d) $v = \frac{mg}{k}(1 - e^{-}kt)$

(vi) A block of mass m resting on a table is attached to a spring of spring constant k and $\omega_0 - \sqrt{\frac{k}{m}}$. The other end of the spring is fixed to the wall. The block is subjected to a frictional force $-2m\beta\dot{x}(\beta < \omega_0, \beta > 0)$ and is released from rest from the position $x = A$ Which of the following is a possible correct solution for the subsequent motion of the block ?

(a) $Ae^{-\beta t}e^{-\sqrt{\omega_0^2-\beta^2}t}$
(b) $Ae^{-\beta t}e^{\sqrt{\omega_0^2-\beta^2}t}$
(c) $Ae^{-\beta t}\cos(\sqrt{\omega_0^2 - \beta^2}t)$
(d) $Ae^{-\beta t}\sin(\sqrt{\omega_0^2 - \beta^2}t)$

(vii) A particle with charge q and mass m is moving under the influence of force $q(\mathbf{v} \times \mathbf{B})$ where $\mathbf{v}$ is the velocity of the particle and $\mathbf{B}$ is the magnetic field. Which of the following statements about the motion of the particle is true in general?

(a) Linear momentum is conserved
(b) Kinetic energy is conserved
(c)Angular momentum is conserved.

2.(**Marks** $= \mathbf{3 + 3 + 4 + 4 = 14}$)

A particle P of unit mass moves on the positive x- axis under the force field

$$F = \frac{36}{x^3} - \frac{9}{x^2}$$

where $x > 0$.
(a) Find the potential $U(x)$ corresponding to this force and make a rough plot of $U(x)$ vs x.
(b) Show that the motion of P consists of either(i) periodic oscillation between two extreme points or (ii) an unbounded motion with one extreme point, depending upon the value of total energy.
(c) Initially, P is projected from the point $x = 4$ with speed 0.5. Show that P oscillates between two extreme points and find the period of the motion. You may make use of the formula

$$\int_a^b \frac{xdx}{[(x-a)(b-x)]^{\frac{1}{2}}} = \frac{\pi(a+b)}{2}$$

(d) Show that there is a single equilibrium position for P and that it is stable. Find the period of small oscillations about this point.

3. (**Marks** $= \mathbf{8 + 6 = 14}$)

a) A particle with polar coordinates r, θ which are functions of time t is moving in a plane. The velocity and acceleration of the particle can be written in plane polar coordinates as $\mathbf{v} = v_r\hat{\mathbf{r}} + v_\theta\hat{\theta}$ and $\mathbf{a} = a_r\hat{\mathbf{r}} + a_\theta\hat{\theta}$. Find $v_r, v_\theta, a_r, a_\theta$ as functions of $(r, \theta, \dot{r}, \dot{\theta})$

b) An insect flies on a spiral trajectory such that its polar coordinates at time t are given by $r = be^{\Omega t}, \theta = \Omega t$. where b and Ω are positive constants. Find the velocity and acceleration vectors of the insect at time t and show that the angle between them is always $\frac{\pi}{4}$

4. (**Marks** $= \mathbf{8 + 6 = 14}$)

Consider a projectile fired vertically in a constant gravitational field. (i) For the same initial velocities u, compare the times required for the projectile to reach its maximum height (a)for zero resisting force (b) for a resisting force proportional to the instantaneous velocity of the projectile ($\mathbf{F}_{resistance} = -km\mathbf{v}$, where k is a positive constant, m is the mass of the projectile and $\mathbf{v}$ its velocity.
(ii) Find the maximum height reached by the projectile in the case (b). Find a limiting expression for the above maximum height for the case $\frac{ku}{g} \ll 1$ and hence assure yourself that the maximum height will be reduced by the presence of resistance.

5. (**Marks** $= \mathbf{4 + 6 + 4 = 14}$)

(a) Find the force law for a central force field that allows a particle of mass m to move in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and α are constants.
(b) Find $r(t)$ and $\theta(t)$ for the particle moving with a given angular momentum l which starts out at the origin under the influence of the above force.
(c) What is the total energy of the particle ? (Assume the usual condition $U(r = \infty) = 0$). Does the motion obey Kepler's laws ?

6. (**Marks** $= \mathbf{8 + 6 = 14}$)

An electrical circuit consists of an inductance L, resistance R and a capacitance C connected in series with a battery of emf $\mathcal{E}$. The charge passing through the circuit at a time t is given by $q(t)$ and the current $I(t) = \frac{dq}{dt}$. The parameters are such that $R = 2\sqrt{\frac{L}{C}}$. $q = q_0$ and $I = 0$ at t =0. Kirchoff's equation around the circuit is given by

$$L\frac{dI}{dt} + RI + \frac{q}{C} = \mathcal{E}$$

a) Solve this equation to find $q(t)$. Exploit the analogy between the electrical system and the mass spring system carefully in order to do this.
b) Now remove the resistance from the circuit and find $q(t)$ with the same initial conditions.